Magnetic double refraction in piezoelectrics

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A new type of magneto-optical effect in piezoelectrics is predicted. A low frequency behavior of Faraday effect is found.

The polarization of electromagnetic waves in matter is well known to become circular in magnetic field. However, here I show that in piezoelectrics the Faraday effect is dominated by magnetic double refraction which is related to a new characteristics of matter in the form of a special cross-term tensor.

Let us consider the propagation of low-frequency electromagnetic waves in dielectrics using the principle of least action. The Lagrange variable in electrodynamics is the four-potential $(\varphi, \mathbf{A})$ (see [1]). The electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ are gauge invariant combinations of time and space derivatives of the components of the potential

$$\mathbf{E} = -\nabla\varphi - c^{-1}\dot{\mathbf{A}},\ \mathbf{B} = rot\mathbf{A}.$$

The first two Maxwell equations

$$div\mathbf{B} = 0,\ rot\mathbf{E} = -c^{-1}\dot{\mathbf{B}} \tag{1}$$

are kinematic relations arising from the definitions of $\mathbf{E}$ and $\mathbf{B}$. The second pair of Maxwell equations

$$div\mathbf{D} = 0,\ rot\mathbf{H} = c^{-1}\dot{\mathbf{D}} \tag{2}$$

are dynamic relations coming out from the variational procedure. The fields $\mathbf{D}$ and $\mathbf{H}$ are variational derivatives

$$\mathbf{D} = 4\pi\frac{\delta S}{\delta \mathbf{E}},\ \mathbf{H} = -4\pi\frac{\delta S}{\delta \mathbf{B}} \tag{3}$$

of the action $S = \int L dV dt$. The density of the Lagrange function L is a gauge invariant functional of the four-potential. In order to avoid redundant modes, only first time derivatives of the components of the four-potential have to be taken into account in the Lagrange method. Besides, the kinematic relations (1) must be used to preclude from doubling of Lagrangian terms.

With the amplitude of the electromagnetic field being small the Lagrange function can be expanded in its power series. A similar expansion can be made in the vicinity of some constant field as well.

In the harmonic approximation, the Lagrange function of an isotropic medium is as follows:

$$L = \varepsilon\frac{\mathbf{E}^2}{8\pi} - \frac{\mathbf{B}^2}{8\pi\mu}. \tag{4}$$

Accordingly, we obtain $\mathbf{D} = \varepsilon\mathbf{E}$ and $\mathbf{H} = \mathbf{B}/\mu$, so that ε is the permittivity and μ is the magnetic permeability.

Note, that the speed of electromagnetic waves in a medium $\tilde{c} = c/\sqrt{\varepsilon\mu}$ should be smaller than its value in vacuum c. Otherwise, it leads to a contradiction with the special relativity principle. Consequently, in addition to the usual inequalities $\varepsilon > 1$ and $\mu > 0$ we have: $\varepsilon\mu > 1$.

Generally, magnetic double refraction exists in any material (see [2] §101). In isotropic media, for example, it is described by the following terms of the Lagrange function

$$L_4 = \beta\frac{(\mathbf{EB})^2}{8\pi} + \gamma\frac{(\mathbf{B}^2)^2}{16\pi}. \tag{5}$$

They give anisotropic corrections to the permittivity tensor $\delta\varepsilon_{ik} = \beta B_i B_k$ and to the inverse tensor of the magnetic permeability $\delta\mu^{-1}_{ik} = \gamma(B^2\delta_{ik} + 2B_iB_k)$. However, this quadratic effect is small in comparison with the linear Faraday effect.

At low crystal symmetry the cubic terms can appear:

$$L_3 = \frac{\zeta_{ijk}}{8\pi} B_i B_j E_k. \tag{6}$$

The tensor ζ_{ijk} has the symmetry of a piezoelectric tensor. It determines the linear magnetic double refraction, which is a magnetic analog of the linear Kerr effect. Presumably, this magnetic effect should be observable in the paramagnetic state of materials with strong spin-orbital interactions.

Let us consider this effect in the simplest case of crystal symmetry $\mathbf{T}_d$. There is the only invariant

$$L_3 = \frac{\zeta}{4\pi}(E_xB_yB_z + E_yB_zB_x + E_zB_xB_y), \tag{7}$$

that gives rise to nonlinear terms in the fields:

$$D_x = \varepsilon E_x + \zeta B_y B_z; H_x = \nu B_x - \zeta(E_yB_z + E_zB_y), \tag{8}$$

where $\nu = 1/\mu$. Other components can be obtained by cyclic permutations of the space indices.

Consider an electromagnetic wave of a small amplitude propagating in the presence of constant magnetic field. Using lower case letters for the oscillating fields and upper case letters for the constant field components, we write:

$$d_x = \varepsilon e_x + \zeta(B_yb_z + B_zb_y); h_x = \nu b_x - \zeta(B_ze_y + B_ye_z). \tag{9}$$

Performing Fourier transformation ($\propto e^{-i\omega t + i\mathbf{qr}}$) one can see that the imaginary unit i does not appear in the

coefficients. It means that non-degenerate electromagnetic waves have linear polarization. Using the second Maxwell equation $\omega\mathbf{b} = c[\mathbf{qe}]$ one can write the fourth Maxwell equation as

$$\{\varepsilon\omega^2 - \nu c^2(q_y^2 + q_z^2) + 2c\zeta\omega(B_z q_z - B_y q_y)\}e_x + \quad (10)$$
$$+\{\nu c^2 q_x q_y + c\zeta\omega(B_y q_x - B_x q_y)\}e_y + \quad (11)$$
$$+\{\nu c^2 q_x q_z + c\zeta\omega(B_x q_z - B_z q_x)\}e_z = 0. \quad (12)$$

From this system of equations one can find the spectrum of electromagnetic waves

$$\omega = \left(1 \pm 2\zeta\mu B\sqrt{f}\right)\tilde{c}q, \quad (13)$$

where f is a function of unit vectors $\mathbf{n}$ and $\mathbf{l}$

$$f = (n_x^2 + n_y^2 n_z^2)l_x^2 - 2n_x n_y(1 - n_z^2)l_x l_y + ..., \quad (14)$$

where $\mathbf{n} = \mathbf{q}/|\mathbf{q}|$, $\mathbf{l} = \mathbf{B}/|\mathbf{B}|$, and ... denotes the result of cyclic permutations. The sign in the expression (13) changes after reversing the direction of either field or wave-vector. If $\zeta > 0$, plus corresponds to e_y-wave and minus corresponds to e_x-wave for the field and wave-vector oriented along z-axis [001]. The function f is non-negative and becomes zero if

$$(n_x^2 + n_y^2 n_z^2)l_x = [(1 - 2n_z^2)n_y l_y + (1 - 2n_y^2)n_z l_z]n_x; \dots .$$

It is significant that for any direction of the wave-vector there exists a field orientation when $f = 0$. In its vicinity the Faraday effect dominates over the magnetic double refraction.

Indeed, at low frequency the Faraday effect originates from the following term of the Lagrangian:

$$L_F = \frac{\alpha}{4\pi}E_i(\mathbf{B}\nabla)B_i. \quad (15)$$

For simplicity, only the isotropic term is taken into account. Keeping previous notations, in harmonic approximation we obtain

$$\frac{\alpha}{4\pi}e_i(\mathbf{B}\nabla)b_i. \quad (16)$$

Accordingly, we find

$$\mathbf{d} = \varepsilon\mathbf{e} + \alpha(\mathbf{B}\nabla)\mathbf{b};\ \mathbf{h} = \nu\mathbf{b} + \alpha(\mathbf{B}\nabla)\mathbf{e}. \quad (17)$$

Using the second Maxwell equation one can write the fourth equation as

$$\nu rot\mathbf{b} + 2\alpha(\mathbf{B}\nabla)rot\mathbf{e} = c^{-1}\varepsilon\dot{\mathbf{e}}. \quad (18)$$

For Fourier components we have

$$\nu[\mathbf{qb}] + c^{-1}\varepsilon\omega\mathbf{e} + 2i\alpha(\mathbf{Bq})[\mathbf{qe}] = 0, \quad (19)$$

or, in terms of vector potential $\mathbf{a}$ it reads as follows

$$(\omega^2 - \tilde{c}^2 q^2)\mathbf{a} + 2i\alpha c\varepsilon^{-1}\omega(\mathbf{Bq})[\mathbf{qa}] = 0. \quad (20)$$

Taking into account the smallness of the correction we obtain the spectrum of circularly polarized waves:

$$\omega = \left(1 \pm \alpha\sqrt{\frac{\mu}{\varepsilon}}(\mathbf{Bq})\right)\tilde{c}q, \quad (21)$$

where different signs correspond to the right and left polarization. In the general case, when the function f is of order unity, the correction to the spectrum (21) is smaller ($\propto q^2$) than the correction ($\propto q$) due to the magnetic double refraction effect (13).

One should note that there is an intrinsic limitation in a theory of low frequency modes. Indeed, this approach can only capture a qualitative picture of such phenomena as natural optical activity and Faraday effect. More closely, the multiplier in the correction $\propto q^2$ to the spectrum (21) can be re-normalized if one takes gap modes into account. A similar situation arises in the consideration of the anisotropy of spectra for both electromagnetic waves in cubic crystals and sound waves in the basic plane of hexagonal crystals.

Finally, we see that the cross-term corrections (9), (17) appear in the electric and magnetic responses of the matter. Consequently, the usual assumption ([2], §101) that the theory of electromagnetic waves can be formulated solely in terms of permittivity does not find support.

[1] L.D. Landau, E.M. Lifshitz, The classical theory of fields, Butterworth-Heinemann (1980)

[2] L.D. Landau, E.M. Lifshitz, Electrodynamics of continuous media, Pergamon Press (1984)